Mathematical Development of Tube Loading in Horizontal Condensers

Equations were derived by Nusselt for condensation on a vertical bank of horizontal tubes employing several questionable assumptions. His theoretical results indicate that the average condensing coefficient for a tube in an n-tube vertical bank should be $n^{-1/4}$ times the single-tube coefficient. An empirical modification for turbulence previously suggested by the present author changed the factor to $n^{-1/6}$. To facilitate further experimental studies and design calculations, precise equations are developed for condensate loading for the different common tube layouts bounded by a circle. These equations use a generalized factor $n^{-1/s}$ where a value of $s/4 > 1.0$ becomes an index of turbulence.

DONALD Q. KERN

D. Q. Kern Associates, Cleveland, Ohio

The comparison of performance between large horizontal condensers employing bare and low-finned tubing has amplified several deficiencies in current condenser theory. First, there is no criterion for determining the deviation of the condensate from viscous flow as it descends outside the tubes of a circular tubular condenser. Second, the theoretical average tube loading or disposition of the condensate in viscous flow in a circular tubular condenser has not been described through suitable equations. It is the object of this paper to set up the appropriate viscous-flow equations to enable the measurement of deviations through the correlation of experimental data.

The availability of low-finned tubing as shown in Figure 1 greatly facilitates the further development of condenser theory. Low-finned tubes generally have the same fin outside diameters as conventional condenser tubes. The fin surface is developed by depressing the outside diameter of the bare tube to form interstices and leaving fins 1/16 in. high. For bare tubes the outside surface per linear foot is proportional only to the outside diameter of the tube. With low-finned tubes on the same layout a wide variation in surface and condensate drainage can be obtained without a significant change in the vapor flow distribution. The latter is presumed to be the basis for the attainment of unusually high condensing coefficients on commercial low-finned tubes.

The common method for correlating bare-tube data employs the equation derived by Nusselt (*1*) for condensation on a single horizontal tube with the condensate in viscous flow. Nusselt, employing the same assumptions, also derived an equation for computing the condensing coefficient for any horizontal tube in a vertical bank. For the pth tube from the top the equation for the condensing coefficient is $h_p = h_1[p^{3/4} - (p-1)^{3/4}]$ where h_1 refers to the value of the coefficient computed for a single tube. Nusselt assumed that

1. The resistance to condensation is the resistance of the condensate film to conduction.
2. The temperature difference between the vapor and tube wall is constant.
3. The condensate moves about the tube in viscous flow.
4. The condensate descends from one tube to the tube below as a continuous sheet without disturbing the condensate on the tube below.

Observations on vertical banks of tubes have been made by Young and Wohlenberg (*2*), Katz and Geist (*3*), and Short and Brown (*4*) with banks which were respectively two to five, six and twenty tubes high. It appears from the data of Short and Brown that the first two Nusselt assumptions hold within the range of general condenser operation. The third and fourth assumptions lead to significant deviations when applied to tubes in vertical banks. The condensate does not drain as a continuous sheet. Instead it drains as droplets or, at high loadings, as continuous stream points. Either effect has a disrupting influence on the viscous film concept.

When drainage occurs as droplets, ripples are formed on the bottom of the tube owing to the intermittent dripping action. Ripples are also formed on the tubes on which the droplets impinge. Where there are continuous stream points falling on a lower tube, the condensate does not redistribute to attain a uniform film thickness over the length of the tube but descends about the tube in a veinlike manner. The fluid in the veins attains a degree of turbulence which may locally exceed that associated with viscous flow.

In the comparison between low-finned and bare tubes in vertical banks there are other factors which suggest the need for a refinement in the calculation of average tube loading as well as the correction for the transition from viscous to turbulent flow. The low-finned tubes have natural drainage points at the bottom edges of the fins which are dispersed over the entire length of the tube. Visually there are more droplets of smaller size formed per unit length on low-finned tubes; however, the significance of the drainage can be evaluated only through the correlation of performance data, as the surface per linear foot for tubes of the same nominal diameter differs for both.

At higher loadings and continuous stream points the opportunity for a reduction in the thickness of the vein by a lateral distribution of the condensate on the tube appears to be lessened by the fins. While commercial fins are usually only 0.0625 in. high, the average thickness of the condensate film for an organic liquid with a viscous condensing coefficient of 200 would be about 0.006 in., or only about one tenth as high.

In their analysis of a bank twenty tubes high Short and Brown found that the upper and lower few tubes exceeded the value computed from Nusselt for the top tube alone, where the coefficient should have been highest. They concluded that the performance of the bank as a whole is closely approximated by the top-tube equation rather than the equation for a bank. Katz and Geist employing Freon-12, n-butane, acetone, and water, conducted their experiments on a bank of six low-finned tubes and consistently obtained average coefficients exceeding the Nusselt values for the bank by 121% for steam to 153% for acetone. The bottom tubes of their bank produced condensing coefficients which ranged from 86 to 100% of the top-tube coefficient although the theoretical bottom-tube coefficient should have been only 64% of the top-tube coefficient.

ARRAYS BOUNDED BY CIRCLES

The foregoing has referred to horizontal tubes in single vertical banks for which experimental comparisons are available. A search of the literature does not reveal a comparable mathematical or experimental analysis for the average condensate loading on the tubes of a bundle having vertical tube banks arrayed within a circle.

A representative commercial condenser will consist of a number of vertical banks of tubes of different heights symmetrically disposed about the vertical center line of the shell cross section. In addition, the mechanical design will require the presence of transverse baffles or support plates to damp the harmonics of the tubes. Because of the orientation of the baffles there will be vapor flow across the bundle at right angles to the condensate drainage. All of these, as well as the general decrease in performance of a vertical tube bank below that of the top tube, affect the value of the average condensing coefficient for the bundle.

Kern (*5*) proposed a correction for circular tube bundles for deviations from pure viscous flow which adjusted the condensate loading from its average viscous value. The average loading per tube for a single horizontal row of tubes is given by $G' = W/Ln_t$ where W is the hourly condensation rate, L the tube length, and n_t the total number of tubes. The suggested loading for the condenser with circular cross section was defined by $G'' = W/Ln_t^{2/3}$. Certain tests (*6*) on refrigerants have confirmed the order of magnitude of the exponent.

Derivations

Nusselt's value for the heat transfer coefficient of a single horizontal tube is

$$h_1 = 0.725\left[\frac{k^3\rho^2\lambda}{\mu D(t_f - t_s)}\right]^{1/4} \quad (1)$$

He gives for the effectiveness of the pth tube down in a vertical row

$$h_p = h_1[p^{3/4} - (p-1)^{3/4}] \quad (2)$$

From this it follows that the average coefficient for a bank n tubes high is

$$\bar{h}_n = \frac{1}{n}\sum_{p=1}^{n} h_p = \frac{h_1}{n}\cdot n^{3/4} = h_1 n^{-1/4} \quad (3a)$$

The correction proposed by Kern would change this dependence for the average coefficient to

$$\bar{h}_n = h_1(n^{-1/4})^{2/3} = h_1 n^{-1/6} \quad (3b)$$

Generalized, the relation for the average coefficient may be written

$$\bar{h}_n = h_1 n^{-1/s} \quad (3c)$$

where s has the value of 4 in subsequent references in this paper to the Nusselt theory and 6 for the Kern modification. The value of $s/4$ established by experiments thus becomes an index of the extent by which the condenser deviates from pure viscous flow and assumes partial turbulence.

1. *Square Pitch.* The circular shell of Figure 2 is considered as having an outer tube limit* of radius R containing an array of tubes on square layout with a spacing equal to the tube pitch P_T. The principal axis will be the vertical diameter. If it is assumed that the number of banks along the horizontal diameter is odd and numbered from the center to the right, 0, 1, 2, etc., l_i will be the length of the chord through the ith bank.

$$l_i^2 = 4(R^2 - i^2P_T^2) \quad (4)$$

Then the number of tubes n_i in the ith bank is

$$n_i = l_i/P_T = 2\alpha(1 - i^2/\alpha^2)^{1/2} \quad (5)$$

where $\alpha = R/P_T$. The total number of tubes in the entire array is expressed by

$$n_t = 2\sum_{i=0}^{\alpha-1} n_i - n_0 = 4\alpha\sum_{i=0}^{\alpha-1}(1 - i^2/\alpha^2)^{1/2} - 2\alpha \quad (6)$$

The sum which appears in Equation (6) is evaluated with the aid of the Euler-Maclaurin expansion (*7*) as follows:

$$\sum_{p=0}^{n} f(p) = \int_0^n f(x)\,dx + \frac{1}{2}f(0) + \frac{1}{2}f(n) + \frac{1}{12}[f'(n) - f'(0)] + \cdots \quad (7a)$$

On evaluation,

$$\sum_{i=0}^{\alpha-1}(1 - i^2/\alpha^2)^{1/2}$$

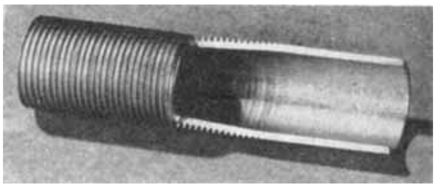

Fig. 1. Low fins extruded from a bare tube provide 2-1/2 times as much condensing surface (*Courtesy of Wolverine Tube*).

It will be noted that

$$f(x) = (1 - x^2/\alpha^2)^{1/2} \quad (7b)$$

$$f'(x) = -\frac{x}{\alpha^2}(1 - x^2/\alpha^2)^{-1/2} \quad (7c)$$

$$n = \alpha - 1 \quad (7d)$$

$$\int_0^n f(x)\,dx = \int_0^{\alpha-1}(1 - x^2/\alpha^2)^{1/2}\,dx = \alpha f_{1/2}(1/\alpha) \quad (7e)$$

Introducing the gamma function (*8*) to evaluate $\alpha f_{1/2}(1/\alpha)$ and inserting into Equation (6) gives for the number of tubes

$$n_t = \pi\alpha^2[1 - 0.3745\alpha^{-3/2} + 0.191\alpha^{-5/2}] \quad (8)$$

The average heat transfer coefficient for a single tube in the entire array is

$$\bar{h} = \frac{1}{n_t}\left[2\sum_{i=0}^{\alpha-1} n_i\bar{h}_{n_i} - n_0\bar{h}_{n_0}\right] = \frac{h_1}{n_t}\left[2\sum_{i=0}^{\alpha-1} n_i^{(s-1)/s} - n_0^{(s-1)/s}\right] \quad (9)$$

since Equation (3) gives $\bar{h}_{n_i}$. Using the value of n_i in Equation (5) gives

$$\bar{h} = \frac{h_1}{n_t}(2\alpha)^{(s-1)/s}\left[2\sum_{i=0}^{\alpha-1}(1 - i^2/\alpha^2)^{(s-1)/2s} - 1\right] \quad (10)$$

On evaluation,

$$\sum_{i=0}^{\alpha-1}(1 - i^2/\alpha^2)^m,$$

where $m = \dfrac{s-1}{2s}$, Equation (7*a*) can be used with

$$f(x) = (1 - x^2/\alpha^2)^m \quad (11a)$$

$$f'(x) = -\frac{2mx}{\alpha^2}(1 - x^2/\alpha^2)^{m-1} \quad (11b)$$

$$\int_0^n f(x)\,dx = \int_0^{\alpha-1}(1 - x^2/\alpha^2)^m\,dx = \alpha f_m(1/\alpha) \quad (11c)$$

Substituting into Equation (7*a*) and evaluating $\alpha f_m(1/\alpha)$ as before gives for the average heat transfer coefficient of the entire array

$$\bar{h} = 2^{2m+1}\alpha^{2m+1}\frac{h_1}{n_t}[a_m + b_m\alpha^{-m-1} + c_m\alpha^{-m-2}] \quad (12a)$$

where

$$a_m = \frac{\sqrt{\pi}\,\Gamma(m+1)}{2\Gamma(m+3/2)} \quad (12b)$$

$$b_m = 2^{m-1}\left[\frac{m-1}{m+1} - \frac{m}{6}\right] \quad (12c)$$

$$c_m = 2^m m\left[\frac{1}{m+2} + \frac{m-5}{24}\right] \quad (12d)$$

Using for n_t the value found in Equation (8) gives

$$\bar{h} = \frac{2^{2m+1}a_m}{\pi}\alpha^{2m-1}\left[1 + \frac{b_m}{a_m}\alpha^{-m-1} + 0.3745\alpha^{-3/2}\right] \quad (13a)$$

For $s = 4$ and $m = 3/8$

$$\bar{h} = 0.885\alpha^{-1/4}h_1\cdot[1 - 0.406\alpha^{-11/8} + 0.3745\alpha^{-3/2}] \quad (13b)$$

For $s = 6$ and $m = 5/12$

$$\bar{h} = 0.922\alpha^{-1/6}h_1\cdot[1 - 0.395\alpha^{-17/12} + 0.3745\alpha^{-3/2}] \quad (13c)$$

*The outer tube limit (O.T.L.) is defined as the diameter of the circle which may not be intercepted by tubes when they are laid out on a tube sheet. In a 12-in. I.D. shell designed for inside-floating-head construction the O.T.L. will be 10-¾ in. and in a 42-in. I.D. shell it will be 40-¾ in.

2. *Square Pitch Rotated by 45 Deg.* As shown in Figure 3, the spacing between vertical banks of tubes is now $P_T/\sqrt{2}$ and the spacing between tubes in a bank is $P_T\sqrt{2}$. The length of the chord through the ith bank from the center is

$$l_i^2 = 2\left(R^2 - \frac{i^2}{2}P_T^2\right)^{1/2} \tag{14}$$

The number of tubes in the ith bank is

$$n_i = l_i/P_T\sqrt{2} = (2\alpha^2 - i^2)^{1/2} \tag{15}$$

The number of tubes in the entire array is

$$n_t = 2\sum_{i=0}^{\alpha\sqrt{2}-1} n_i - n_0 = 2\beta\sum_{i=0}^{\beta-1}(1 - i^2/\beta^2)^{1/2} - \beta \tag{16}$$

where $\beta = \alpha\sqrt{2}$. Evaluated by the methods used in section 1,

$$n_t = \pi\alpha^2[1 - 0.223\alpha^{-3/2} + 0.081\alpha^{-5/2}] \tag{17}$$

This value differs from that of Equation (8) by a leading term of $0.151\alpha^{-3/2}$, a negligible discrepancy. (Rotation by 45 deg. should not change the number of tubes in an array, only the relationship between the number of vertical and horizontal rows.) The average heat transfer coefficient for this array is

$$\bar{h} = \frac{1}{n_t}\left[2\sum_{i=0}^{\alpha\sqrt{2}-1} n_i\bar{h}_{n_i} - n_0\bar{h}_{n_0}\right] = \frac{h_1}{n_t}\left[2\sum_{i=0}^{\beta-1} n_i^{(s-1)/s} - n_0^{(s-1)/s}\right] \tag{18}$$

Again employing the methods of section 1 yields

$$h = 2\beta^{(s-1)/s}\frac{h_1}{n_t}[a_m\beta + b_m\beta^{-m} + c_m\beta^{-m-1}] \tag{19}$$

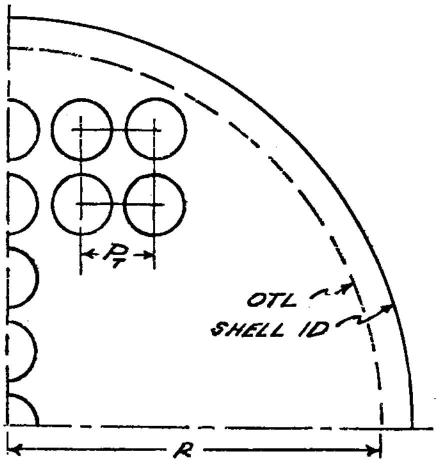

Fig. 2. Square pitch.

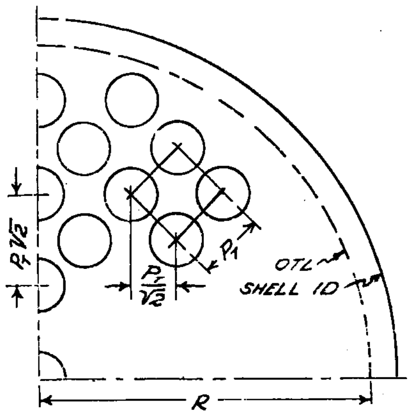

Fig. 3. Square pitch rotated 45 deg.

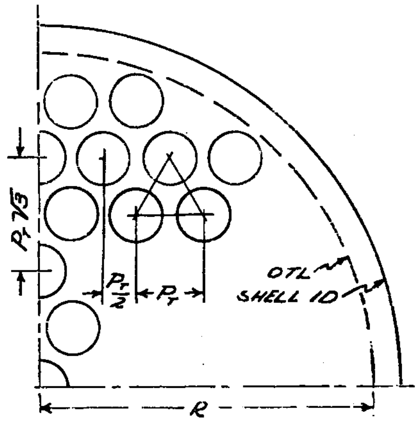

Fig. 4. Triangular pitch, vertex up.

and inserting the values for β and n_t gives

$$\bar{h} = \frac{2^{m+3/2}a_m}{\pi}\alpha^{2m-1}h_1\left[1 + 2^{(-m-1)/2}\frac{b_m}{a_m}\alpha^{-m-1} + 0.223\alpha^{-3/2}\right] \tag{20a}$$

For $s = 4$ and $m = 3/8$

$$\bar{h} = 0.972\alpha^{-1/4}h_1[1 - 0.251\alpha^{-11/8} + 0.223\alpha^{-3/2}] \tag{20b}$$

For $s = 6$ and $m = 5/12$

$$\bar{h} = 0.975\alpha^{-1/6}h_1 \cdot [1 - 0.242\alpha^{-17/12} + 0.223\alpha^{-3/2}] \tag{20c}$$

Comparison of Equations (13*a*) and (20*a*) shows that for large arrays on square pitch, rotation from the vertical by 45 deg. should increase the theoretical average heat transfer coefficient by $2^{-m+1/2}$.

For $s = 4$ this factor is $2^{1/8}$, or 1.091, and for $s = 6$ it is $2^{1/12}$, or 1.075.

3. *Equilateral Triangular Pitch, Vertex Up.* As shown in Figure 4, the spacing between vertical rows of tubes is $P_T/2$ and between tubes in a vertical row $P_T\sqrt{3}$. The length of the chord through the ith bank is

$$l_i = 2\left[R^2 - \frac{i^2}{4}P_T^2\right]^{1/2} = 2P_T\left[\alpha^2 - \frac{i^2}{4}\right]^{1/2} \tag{21}$$

The number of tubes in the ith bank is

$$n_i = \frac{l_i}{P_T\sqrt{3}} = \frac{2}{\sqrt{3}}\left(\alpha^2 - \frac{i^2}{4}\right)^{1/2} \tag{22}$$

The total number of tubes in the array is

$$n_t = 2\sum_{i=0}^{2\alpha} n_i - n_0 = \frac{4\alpha}{\sqrt{3}}\left[\sum_{i=0}^{2\alpha}(1 - i^2/4\alpha^2)^{1/2} - \frac{1}{2}\right] = \frac{2}{\sqrt{3}}\pi\alpha^2 \tag{23}$$

There are thus $2/\sqrt{3} = 1.152$ times as many tubes in a given triangular pitch as in the same square pitch for shells of identical radii. The average heat transfer coefficient for the array is

$$\bar{h} = \frac{1}{n_t}\left[2\sum_{i=0}^{2\alpha} n_i\bar{h}_{n_i} - n_0\bar{h}_{n_0}\right] = \frac{h_1}{n_t}\left[2\sum_{i=0}^{2\alpha} n_i^{(s-1)/s} - n_0^{(s-1)/s}\right] \tag{24}$$

Solving as in sections 1 and 2 gives

$$\bar{h} = \frac{4}{\pi}\left(\frac{2}{\sqrt{3}}\right)^{2m-1}a_m\alpha^{2m-1}h_1 \tag{25a}$$

For $s = 4$, this gives

$$\bar{h} = 1.070\alpha^{-1/4}h_1 \tag{25b}$$

and for $s = 6$,

$$\bar{h} = 1.010\alpha^{-1/6}h_1 \tag{25c}$$

U-TUBE ARRAYS BOUNDED BY CIRCLES

Use of U-bend tubes requires that the two horizontal rows of tubes which straddle the horizontal diameter be spaced apart by a minimum of two pitches, rather than one pitch as formerly, because deformation of the tube cross section is reduced during bending when the minimum bend has a diameter not less than 2.5 times the tube outside diameter. Since it is fairly common practice (*9*) to employ a pitch 1.25 times the tube outside diameter, it follows that two tube pitches will fill this requirement.

1. *Square Pitch.* For this array, $2\alpha - 1$ tubes are subtracted, and

$$n_t = \pi\alpha^2 - 2\alpha - 1.178\alpha^{1/2} + 1 \quad (26)$$

is left.

2. *Square Pitch Rotated.* Every second bank loses one tube, and the intermediate banks lose 1.5 tubes closely on the average. Since there are $2\alpha\sqrt{2} - 1$ banks, this deducts about 1.25 $(2\alpha\sqrt{2} - 1)$, or $3.5\alpha - 1$, tubes, leaving

$$n_t = \pi\alpha^2 - 3.5\alpha - 0.701\alpha^{1/2} + 1 \quad (27)$$

3. *Triangular Pitch.* The same reasoning holds, although the loss of the intermediate banks will be less, 1.35 tubes approximately, and the factor is $(1.35 + 1)/2 = 1.18$. There are $4\alpha - 1$ banks; hence the deduction is 1.18 $(4\alpha - 1)$, or $4.7\alpha - 1$, leaving

$$n_t = \frac{2}{\sqrt{3}}\pi\alpha^2 - 4.7\alpha + 1 \quad (28)$$

CONCLUSIONS

The number of tubes within a given outer tube limit is defined by Equations (8), (17), and (23) and will obviously be greater than that contained in manufacturers' tube counts. No allowance has been made for the reduction in the total number of tubes owing to tube-pass orientation, pass partitions, or the usual provision for entry and exit space adjoining the inlet and outlet nozzles. The derivations also consider the center vertical tubes to be located on the vertical center line and not straddling it; however, it can be used with considerable reliability on large condensers for obtaining the approximate tube count for any new equilateral layout.

Equations for the heat transfer coefficient, Equations (13), (20), and (25), lead to certain theoretical conclusions. From the standpoint of viscous flow the optimum use of a horizontal condenser will be derived when it contains a larger number of vertical rows than there are tubes in the largest vertical row. This in turn would discourage the use of equilateral pitch and would favor such arrangements as diamond pitch with a vertical major diagonal.

The solution for triangular pitch having a horizontal altitude has not been undertaken, as this layout would produce a markedly lower theoretical condensing coefficient than that of a triangle having the vertex up and centered. Should a condenser be installed with a horizontal altitude, its performance can be improved at light loading if a minor rotation of the bundle is feasible.

As an illustration of the use of these equations, a circular horizontal condenser with an outer tube limit 30 in. in diameter and tubes on 1-in. square pitch will be considered. The number of tubes computed for the bundle by Equation (8) will be 702, and the average coefficient by Equation (13*b*) will be 45% of the single-tube value computed by Equation (1) and would be 59% according to Equation (13*c*). With the pitch rotated 45 deg. there will be the same number of tubes but the coefficient by Equation (20*b*) will be 49% of the single-tube value; by Equation (20*c*) the average coefficient will be 62% of that value. For 1-in. triangular pitch the computed count by Equation (23) will be 808 tubes, and the average coefficient is 54% of the single-tube value by Equation (25*b*) and 64% by Equation (25*c*).

In the correlation of experimental data it is now possible to determine the value of s by using the appropriate equation for a given layout. Then values of $s/4 > 1$ will serve as an index of turbulence for the condensate. It is doubtful that s will remain constant with increasing condenser diameter, because of the inherent geometry of a circular array, or over a very wide range of tube loadings, as at lighter loading the condensate may approach viscous flow.

NOTATION

D = outside diameter of tube, ft.
G' = average tube loading for condensation on a horizontal bank, lb./hr./ft.
G'' = tube loading for condensation in a circular condenser, lb./hr./ft.
h_1 = condensing coefficient computed for a single tube, B.t.u./(hr./sq. ft./°F.)
$\bar{h}$ = average condensing coefficient for an array of tubes, B.t.u./(hr./sq. ft./°F.)
h_{n_i} = average condensing coefficient for the i bank in a circular condenser, B.t.u./(hr./sq. ft./°F.)
h_{n_o} = average condensing coefficient for the center vertical bank of a circular condenser, B.t.u./(hr./sq. ft./°F.)
h_p = condensing coefficient for the p tube from the top in a vertical bank, B.t.u./(hr./sq. ft./°F.)
i = number of a row to left or right of the center vertical row in a circular condenser
k = thermal conductivity, B.t.u./(hr./sq. ft./°F.)/ft.
L = tube length in a horizontal condenser, ft.
l_i = length of chord through the i vertical row of a condenser, ft.
n = total number of tubes in a vertical row
n_i = number of tubes in the i vertical row in a circular condenser
n_0 = number of tubes in the center vertical row of a circular condenser
n_t = total number of tubes in a circle (O.T.L.)
p = number of a tube in a vertical tube bank counting from the top, or number of tubes in the bank
P_T = tube pitch, ft.
R = radius of a circle which is the outer tube limit for laying out tubes in a shell, ft.
s = exponent, 4 in the Nusselt theory, 6 in the Kern modification
t_f = film temperature, $(t_s + t_w)/2$, °F.
t_s = saturation temperature of vapor, °F.
t_w = tube-wall temperature, °F.
W = condensation rate, lb./hr.
x = variable

Greek Letters

α = ratio of *radius* of outer tube limit to tube pitch
β = $\alpha\sqrt{2}$
λ = latent heat of vaporization, B.t.u./lb.
μ = viscosity, lb./(ft./hr.)
ρ = Density, lb./cu. ft.

ACKNOWLEDGMENT

The author wishes to express his appreciation to the Wolverine Tube Division of Calumet and Hecla, Inc., for permission to publish this portion of the application engineering file.

LITERATURE CITED

1. Nusselt, W., *Z. Ver. deut. Ingenieurw.* **60**, 541 (1916).
2. Young, F. L., and W. J. Wohlenberg, *Trans. Am. Soc. Mech. Engrs.* **64**, 787 (1942).
3. Katz, D. L., and J. M. Geist, *ibid.*, **70**, 907 (1948).
4. Short, B. E., and H. E. Brown, *Proc. Genl. Discussion on Heat Transfer*, sec. I, p. 27, Inst. Mech. Engrs. and Am. Soc. Mech. Engrs., London (1951).
5. Kern, D. Q., "Process Heat Transfer," p. 266, McGraw-Hill Book Company, Inc., New York (1950).
6. Chamberlain, J. R., York Corporation, private communication (1956).
7. Jefferys, Harold, and B. S. Jeffreys, "Methods of Mathematical Physics," 3 ed., p. 279, Cambridge Univ. Press, Cambridge (1956).
8. Jahnke Eugene, and Fortz Emde, "Tables of Functions," p. 14, Dover Publications, Inc., New York (1945).
9. "Standards of Tubular Exchanger Manufacturers Association," 3 ed., **8**, **25**, **49**, New York (1952).

Manuscript received May 9, 1957; revision received Oct. 31, 1957; paper accepted Jan. 20, 1958.